Proposing A Mathematical Model, With Considering Three Fleets Of Vehicles, On-foot Porters And Drones

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Abstract

Due the importance of the on time servicing to customers, the need for creative and innovative mathematical modeling that routes the delivery fleet and minimize the whole time of delivery, is felt more than ever. For example, delivering parceles to customers, can be done by vans, on_foot porters or drones that each of these modes, has its own advantages and disadvantages. Also,some models have been invented for cooperation of vans with drones or on_foot porters with vans, but none of them have yet tried to combine all three modes at the same time. Therefore in this paper, the simultaneous combination of all three mentioned modes that selects the best mode for the delivery of parcels, is suggeste. Also, by comparing our mathematical model with the model given in this paper's base article, it can be logically understood that due to the greater flexibility of our model in choosing the fleet to serve each customer the answer of our model can be equal to or better than the answer of this reference and will not be worse than that.

Keywords: Urban Logistic, Fleet Selection, Vehicle Routing Problems, Developmental And Integrational Model, Triple Relationship, Van, On_Foot Porter, Drone

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Introduction

One of the most important foelds that has been paid attention in thendistribution systems, is delivery in the last mile, which according to the speech of the [1], “Every on, can delivers every where” and for this field, two issues can be stated. The first, is what is our objective function in mathematical modeling such problems that can be optimizing the total delivery time or delivery cost. The second issue, is the which fleet to choose for achieve our desired goals (one or a combination of vans, on_foot porters, drones and etc fleets to achieve our goals), but although researchers have made significant progress in both issues, the combination of three fleets together, has not been investigated and they have not gone beyond the framework of dual fleet cooperation. Therefore in this paper with the help of the courage and creativity that every industrial engineer should be have, for the first time, hase been suggested the simultaneous combination of three fleets(vans, on_foot porters and drones) to routing the delivery vehicles and named it VRPFpD(vehicle routing problems with on_foot porters and drones) and suggested a mathematical model for this combination, inspired by the model in the base reference[2].

The remainder of the paper is structured as follows. First, the literature review of this field and the existing researches, are discussed. In the second part, problem definition and the suggested mathematical model for VRPFpD is explained.

literature review

Last-mile or last-mile delivery, refers to the last leg an item makes before reaching its final destination or consumption point[3] and the cost of last-mile deliveries usually accounts for 41% of total e-commerce supply chain costs[3] .Therefore, the optimization of delivery vehicle routing, can leads to significant savings in the financial systems of parcels distributor companies.

Although many articles have been written in the field of cooperation between drones and vehicles in distribution systems and last_mile delivery that generally this cooperation is called Vehicle Routing Problem with Drones(VRPD), but this cooperation, itself is divided into many sub-sections which among them, Flying Sidekick Traveling Salesman Problem(FSTSP) and Parallel Drone Scheduling Traveling Salesman Problem (PDSTSP) can be mentioned in the figure 1.

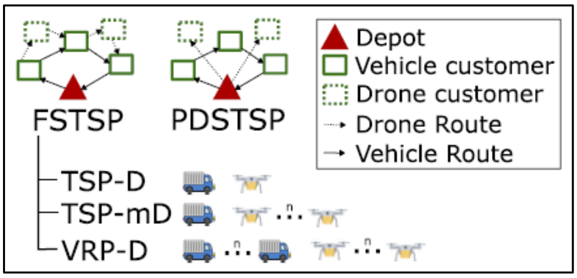

Figure (1) differente between FSTSP and PDSTSP[N.M]

In another researches, the mathematical models have been introduced the relationship between the vehicles and the porters, and in the presence point of each customer, it is decided that the customer's order will be delivered by a vehicle or a porter which if each point becomes the place of exchange of parcels from the vehicle to the porter, it is called the Handover point. So, in [4] researches, it is assumed that each porters returns to its handover pount after passing his delivery route, but the vehicles, are not assumed to wait at each delivery point for the return of the porters. Also, in the model presented by [2], this waiting is not assumed and it is assumed that every porter, does not return to her place after passing the her delivery route, and a summary of the characteristics of other reviewed sources, such as the year of publication, the objective function and the type of solution of the stated model, is also given in Table 1.

Now, the advantages and limitations of each of the three previously mentioned fleets modes, will be discussed. For example, from the point of view of transportation capacity, vehicles have the highest capacity, but in urban areas and especially places that require compliance with environmental or traffic laws, they are less attractive than the other two fleets. Although porters create high maneuverability and flexibility in the distribution system and are not limited by driving rules and regulations or restrictions on crossing the roads, they have a limit on the length of the route and need to rest. They also have less carrying capacity. Also, drones currently have the lowest carrying capacity compared to the other two fleets, but according to the researchs and models that presented by [5], in situations such as delivering parcels to customers in multi-story buildings or with courtyards, the speed of operation cans higher than the two other fleets; but they have limitations such as landing location, variable fuel and charging station.

Therefore, according to the stated reviews for the first triple combination of van, on-foot porters and drone together, the mathematical model that presented with [2], has been chosen as the base model of this paper.

Table 1-Related Workes To This Article

Reference	Publication Year	model		Solution Type	
		Obj Function	Transportation Mode	Exact	Heuristic
[6]	2019	Delivery cost	Vans & cargo bikes	☑	☑
[7]	2020	Delivery time	Vans & walking	☑	☒
[8]	2021	Delivery time	Trucks & drones	☑	☒
[2]	2022	Delivery time	Vans & Portering	☑	☑
[4]	2023	Delivery time	Self_driven cars & drones	☑	☑
This study	-	Delivery time	Vans, Portering & drones	☑	☒

problem definition and mathematical model

One of the most important goals of researchers and distributing companies, is to deliver parcels and customer orders in the shortest time, at the lowest cost and with the best combination of available fleets. but so far, mathematical modeling for combining more than two types of fleets, has not been provided. Also, considering the existence of many mathematical models for the routing of the delivery fleet that assum one or two types of fleet, logically, if the mathematical model can be written in such a way that the fleet selection for each customer point was allowed, the model will selects the best fleet and according to the input parameters of the model and its objective function, the stated goals can be easily achieved. Also in figure 2, can see a view of the concept of the fleet selection that we have considered.

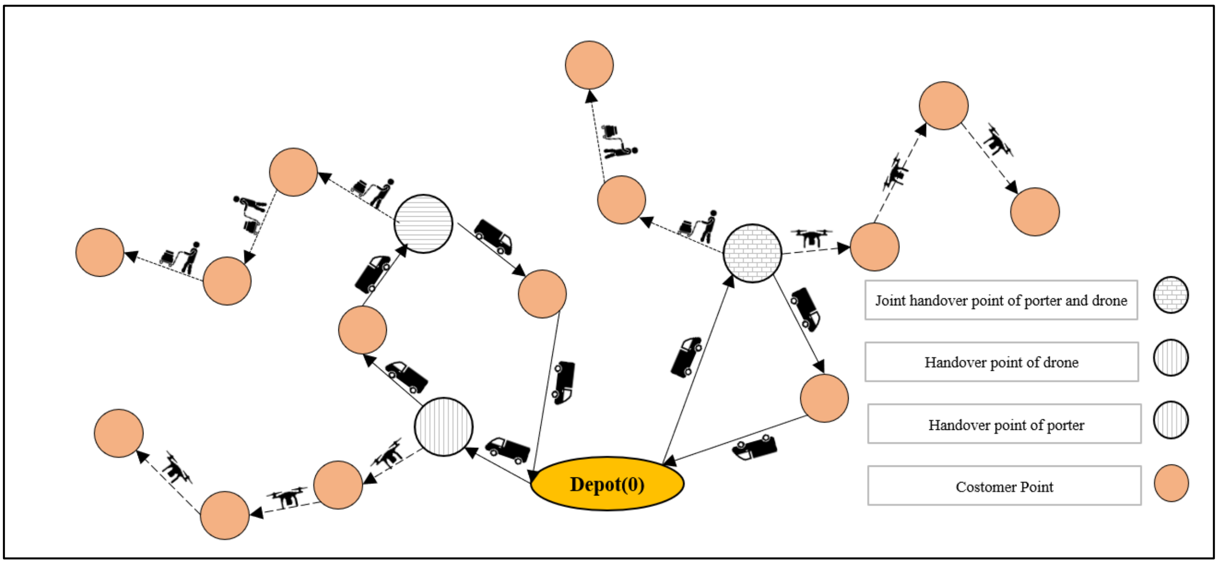

Figure (2) A view of the fleet selection concept in this article

Now the suggested mathematical model for this goal, is discussed and before that, the list of sets, parameters and variables that used in the model can be seen in Table 2. Also in this table, the word "node" is used instead of the word "customer".

Table 2- list of sets, parameters and variables that used in the model

Sets and Indexes	
$\boldsymbol{V}$	Set of costomers nodes and depot
$\boldsymbol{V_c}$	Set of costomers nodes

$\boldsymbol{i, j, k}$	Indexes of costomers nodes
$\boldsymbol{s}$	A dummy node for porters rout
$\boldsymbol{s'}$	A dummy node for drones rout
$\boldsymbol{0}$	Depot index
Parameters and Constants	
$\boldsymbol{t^v_{ij}}$	Travel time from node i to node j with vehicle
$\boldsymbol{t^p_{ij}}$	Travel time from node i to node j with on_foot portering
$\boldsymbol{t^d_{ij}}$	Travel time from node i to node j with drone
$\boldsymbol{N^A_v}$	Availabe number of vehicles
$\boldsymbol{N^A_p}$	Availabe number of on_foot porters
$\boldsymbol{N^A_d}$	Availabe number of drones
$\boldsymbol{d_j}$	Demand of node j $\in V_c$
$\boldsymbol{K_p}$	Capacity of on_foot porterd
$\boldsymbol{K_d}$	Capacity of drones
$\boldsymbol{M}$	A large and constant number (Big M)
Variables	
$\boldsymbol{V_{ij}} \in \mathbb{B}$	1 if goes from node I $\in V$ to node j $\in V_c$ white vehicle, otherwise is 0
$\boldsymbol{P_{ij}} \in \mathbb{B}$	1 if goes from node I $\in V$ to node j $\in V_c$ white On_foor portering, otherwise is 0
$\boldsymbol{D_{ij}} \in \mathbb{B}$	1 if goes from node I $\in V$ to node j $\in V_c$ white drone, otherwise is 0
$\boldsymbol{I^v_{ij}} \in \mathbb{R}^+$	Vehicle inventory while passing from node I $\in V$ to node j $\in V_c$
$\boldsymbol{I^p_{ij}} \in \mathbb{R}^+$	On_foor porter inventory while passing from node I $\in V$ to node j $\in V_c$
$\boldsymbol{I^d_{ij}} \in \mathbb{R}^+$	drone inventory while passing from node I $\in V$ to node j $\in V_c$
$\boldsymbol{D^p_j} \in \mathbb{R}^+$	Total Amount of parcels passing through handover point $k \in Vc$ whith portering
$\boldsymbol{D^d_j} \in \mathbb{R}^+$	Total Amount of parcels passing through handover point $k \in Vc$ whith drones
$\boldsymbol{H^p_{kj}} \in \mathbb{B}$	1 if in linkage frome node k $\in V_c$ to node j $\in V_c$, node k was a handoverpoint for porter, otherwise is 0
$\boldsymbol{H^d_{kj}} \in \mathbb{B}$	1 if in linkage frome node k $\in V_c$ to node j $\in V_c$, node k was a handoverpoint for drone, otherwise is 0
$\boldsymbol{N^p_k} \in \mathbb{Z}^+$	Total number of porters leaving from handover point $k \in Vc$
$\boldsymbol{N^d_k} \in \mathbb{Z}^+$	Total number of drones leaving from handover point $k \in Vc$

Now, the suggested mathematical model is described below; but before that, the assumptions of the following model are mentioned. In the other word, each customer can only be visited by one fleet and one type of fleet. Also to create simplicity in this model and to better understand its concept, it is assumed that the porters and drones, go to the dummy node after traveling their route, do not return to their handover and the vehicles, are allowed to input and output from depot only.

$$\text{Min} \sum\nolimits_{i,j\in V\ ,i\neq j} \boldsymbol{t^v_{ij} V_{ij}} + \sum\nolimits_{i\in V_c \cup \{s\}, i\neq j} \boldsymbol{t^p_{ij} P_{ij}} + \boldsymbol{t^d_{ij} D_{ij}} \tag{1}$$

Subject to:

Vehicles constraints:

$$\sum_{j\in V_c} \boldsymbol{V_{0j}} \leq \boldsymbol{N^A_v} \tag{2}$$

$$\sum_{j\in V\ ,j\neq k} \boldsymbol{V_{jk}} - \sum_{i\in V\ ,i\neq k} \boldsymbol{V_{kj}} = \boldsymbol{0} \qquad \forall k \in V \tag{3}$$

$$\sum_{i\in V\ ,i\neq j} \boldsymbol{I_{ij}^{v}} - \sum_{i\in V\ ,i\neq j} \boldsymbol{I_{ji}^{v}} = \boldsymbol{D_j^{p}} + \boldsymbol{D_j^{d}} + \boldsymbol{d_j} \qquad \forall j \in V_c \qquad (4)$$

$$\sum_{i\in V_c} \boldsymbol{I_{0i}^{v}} - \sum_{i\in V_c} \boldsymbol{d_i} = \boldsymbol{0} \qquad (5)$$

$$\boldsymbol{I_{ij}^{v}} \le (\boldsymbol{K_v} - \boldsymbol{d_i})\, \boldsymbol{V_{ij}} \qquad \forall i \in V_c\,, j \in V\,, i \neq j \qquad (6)$$

$$\boldsymbol{I_{ij}^{v}} \ge \boldsymbol{d_i}\, \boldsymbol{V_{ij}} \qquad \forall i \in V\,, j \in V_c\,, i \neq j \qquad (7)$$

Porters Constraints:

$$\sum_{k\in V_c} \boldsymbol{N_k^{p}} \le \boldsymbol{N_p^{A}} \qquad (8)$$

$$\sum_{j\in V_c\,,j\neq k} \boldsymbol{H_{kj}^{p}} \ge \boldsymbol{N_k^{p}} \qquad \forall k \in V_c \qquad (9)$$

$$\boldsymbol{H_{kj}^{p}} \le \boldsymbol{N_k^{p}} \qquad \forall k, j \in V_c \qquad (10)$$

$$\sum_{i\in V_c\,,i\neq k} \boldsymbol{P_{ki}} - \sum_{i\in V_c\,,i\neq k} \boldsymbol{P_{ik}} = \boldsymbol{N_k^{p}} \qquad \forall k \in V_c \qquad (11)$$

$$\sum_{i\in V_c\,,i\neq j} \boldsymbol{I_{ijk}^{p}} - \sum_{i\in V_c \cup \{s\},i\neq j} \boldsymbol{I_{ijk}^{p}} = \boldsymbol{d_j}\, \boldsymbol{H_{kj}^{p}} \qquad \forall j, k \in V_c \qquad (12)$$

$$\sum_{i\in V_c\,,i\neq j} \boldsymbol{I_{ijk}^{p}} - \sum_{j\in V_c\,,j\neq k} \boldsymbol{d_j}\, \boldsymbol{H_{kj}^{p}} = \boldsymbol{0} \qquad \forall k \in V_c \qquad (13)$$

$$\boldsymbol{I_{ijk}^{p}} \le \boldsymbol{K_p P_{ij}} \qquad \forall i, j, k \in V_c\,, i \neq j \qquad (14)$$

$$\boldsymbol{D_k^{p}} = \sum_{j\in V_c} \boldsymbol{d_j}\, \boldsymbol{H_{kj}^{p}} \qquad \forall k \in V_c \qquad (15)$$

$$\boldsymbol{P_{ij}} \le \sum_{k\in V_c} \boldsymbol{H_{kj}^{p}} \qquad \forall i, j \in V_c\,, i \neq j \qquad (16)$$

$$\boldsymbol{P_{ij}} \le \sum_{k\in V_c} \boldsymbol{H_{ki}^{p}} \qquad \forall i, j \in V_c\,, i \neq j \qquad (17)$$

$$\sum_{i\in V_c\,,i\neq j} \boldsymbol{P_{ij}} = \sum_{k\in V_c\,,k\neq j} \boldsymbol{H_{kj}^{p}} \qquad \forall j \in V_c \qquad (18)$$

$$\sum_{k\in V_c\,,k\neq j} \boldsymbol{H_{kj}^{p}} \le \boldsymbol{1} \qquad \forall j \in V_c \qquad (19)$$

$$\boldsymbol{I_{jsk}^{p}} = \boldsymbol{0} \qquad \forall j, k \in V_c \qquad (20)$$

Drones Constraints:

$$\sum_{k\in V_c} \boldsymbol{N_k^{d}} \le \boldsymbol{N_d^{A}} \qquad (21)$$

$$\sum_{j\in V_c\,,j\neq k} \boldsymbol{H_{kj}^{d}} \ge \boldsymbol{N_k^{d}} \qquad \forall k \in V_c \qquad (22)$$

$$H_{kj}^{d} \leq N_{k}^{d} \tag{23}$$

$$\sum_{i \in V_c, i \neq k} D_{ki} - \sum_{i \in V_c, i \neq k} D_{ik} = N_{k}^{d} \quad \forall k \in V_c \tag{24}$$

$$\sum_{i \in V_c, i \neq j} I_{ijk}^{d} - \sum_{i \in V_c \cup \{s\}, i \neq j} I_{ijk}^{d} = d_j H_{kj}^{d} \quad \forall j,k \in V_c \tag{25}$$

$$\sum_{i \in V_c, i \neq j} I_{ijk}^{d} - \sum_{j \in V_c, j \neq k} d_j H_{kj}^{d} = 0 \quad \forall k \in V_c \tag{26}$$

$$I_{ijk}^{d} \leq K_d D_{ij} \quad \forall i,j,k \in V_c, i \neq j \tag{27}$$

$$D_{k}^{d} = \sum_{j \in V_c} d_j H_{kj}^{d} \quad \forall k \in V_c \tag{28}$$

$$D_{ij} \leq \sum_{k \in V_c} H_{kj}^{d} \quad \forall i,j \in V_c, i \neq j \tag{29}$$

$$D_{ij} \leq \sum_{k \in V_c} H_{ki}^{d} \quad \forall i,j \in V_c, i \neq j \tag{30}$$

$$\sum_{i \in V_c, i \neq j} D_{ij} = \sum_{k \in V_c, k \neq j} H_{kj}^{d} \quad \forall j \in V_c \tag{31}$$

$$\sum_{k \in V_c, k \neq j} H_{kj}^{d} \leq 1 \quad \forall j \in V_c \tag{32}$$

$$I_{js'k}^{d} = 0 \quad \forall j,k \in V_c \tag{33}$$

Common Constraints:

$$\sum_{i \in V_c, i \neq j} V_{ij} + \sum_{i \in V_c, i \neq j} P_{ij} + \sum_{i \in V_c, i \neq j} D_{ij} = 1 \quad \forall j \in V_c \tag{34}$$

$$u_i - u_j + MV_{ij} + (M - t_{ij}^{v} + a_j - b_i)V_{ij} \leq M - t_{ij}^{v} \quad \forall i,j \in V_c, i \neq j \tag{35}$$

$$u_i - u_j + MP_{ij} + \left(M - t_{ij}^{p} + a_j - b_i\right)P_{ij} \leq M - t_{ij}^{p} \quad \forall i,j \in V_c, i \neq j \tag{36}$$

$$u_i - u_j + MD_{ij} + (M - t_{ij}^{d} + a_j - b_i)D_{ij} \leq M - t_{ij}^{d} \quad \forall i,j \in V_c, i \neq j \tag{37}$$

$$a_i \leq u_i \leq b_i \quad \forall i \in V_c \tag{38}$$

The objective function (1), minimize the total travel time. Constraint (2) guarantees that the number of routs that connected to the depot, don't exceed from total available vehicles number. Constraint (3) imposes route flow balance for each nodes thad visited by vehicle. Constraint (4) is the vehicle inventory balance between two linked nodes. Constraint (5) guarantees that total output parcels from depot, be equal to total customers demand. Constraints (6) and (7) guarantee that the vehicle inventory in each nodes that visited by it, not exceed from vehicle capacity and be enogh to service that node. constraint (8) guarantee that the total number of parcels that assigned to handover points, dose note exceed of total available portres in the system.constraints (9) guarantee that if node k is not a handover point, can not allowed to assign porters to it and constraint (10), guarantee that if node k is a handover point, at least 1 porters must be assigned to it. Constraint (11) is a balance for porters in handover point k. constraint (12) is inventory balance for each costomers that serviced by on-foot portering. Constraint (13) is inventory flow balance of total on-foot portering routs that assigned to handover point k. constrint (14) guarantees that porter inventory of every on-foot portes, not exceed of them capacity. Constraint (15) calculates the total parcels that transfered in handover point k to porters. Constraints (16) and (17) guarantees that a porter route exists in node k if and only if node k was a handover point. constrint (18) guarantees that the number of portering routes that assigned to handover point k, must be equal to total porters that allowed to exit of handovrt point k. constraint (19) guarantees that each customers, assigned to at most 1 handover point. Constraint (20) guarantees that no porters go back to its handoverhoint. Constraints (21) to (33), are priority liked as constraint (8) to (20) but for drones. constraint (34) guarantees that each costomer servised by only on and one type of fleets. constraints (35) to (37) are the MTZ subtour eliminators and the Constraint (38) is the time window constraint and guarantees that every node, must be visited in time interval $[a_i, b_i]$.

Conclusions

Becouse this paper is a research and develpe base paper, so we did'n solve this model by Cplex, GAMS or other optimization softwardes but according to out base model in reference [2], we ensure our model can be developed for the actual distributors. In other words, this model is a innovated model tht combined 3 modes of distributing togheter for the firs time and don't need to verificate or solve. Also, by comparing this mathematical model with the model given in the reference [2], it can be logically understood that due to the greater flexibility of our model in choosing the fleet to serve each customer, definitely if we solve the our model with the same data of the reference [2], the answer of our model can be equal to or better than the answer of this reference and will not be worse than that.

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